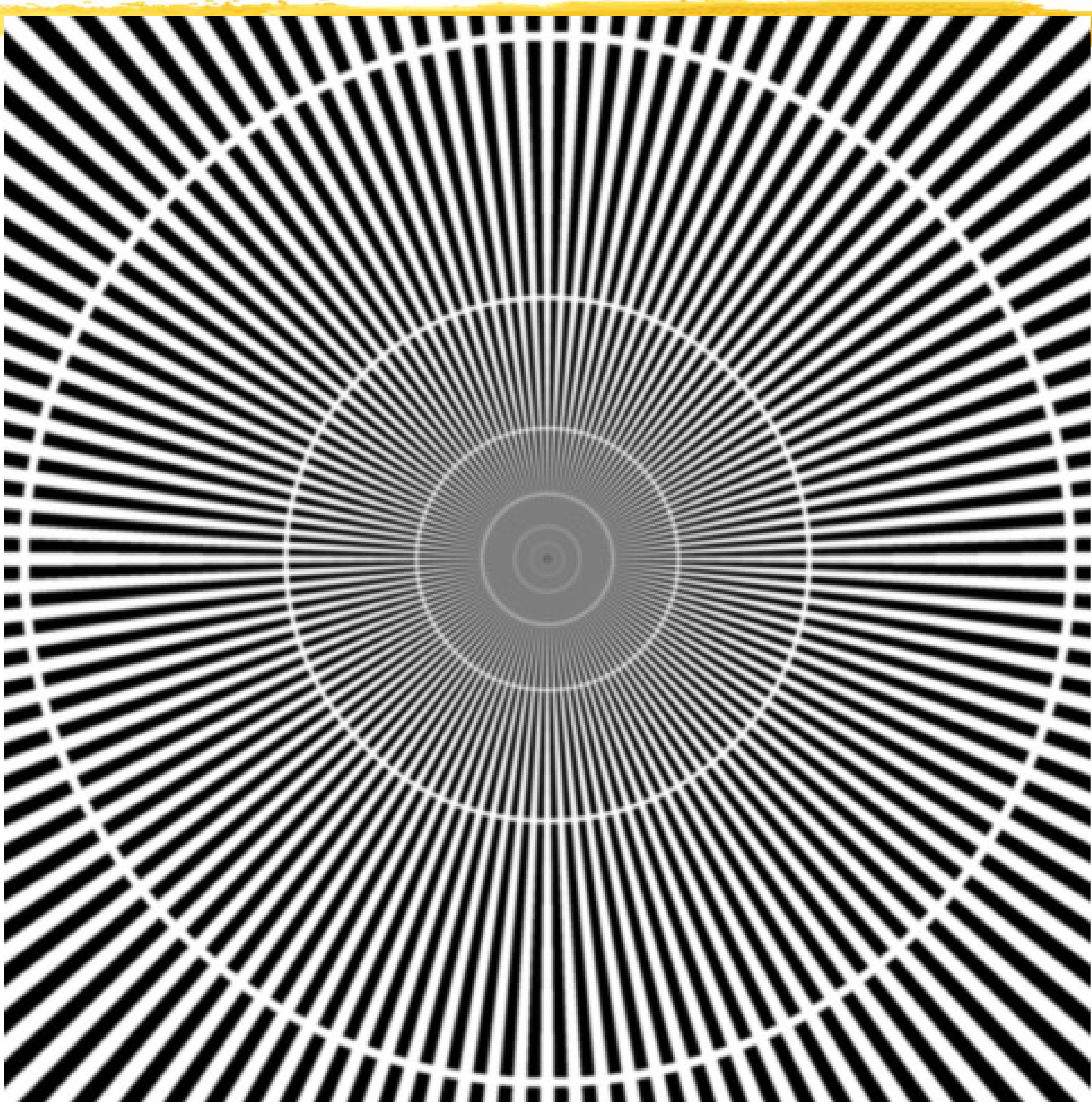
Scan Conversion

- Drawing Lines
- Drawing Circles

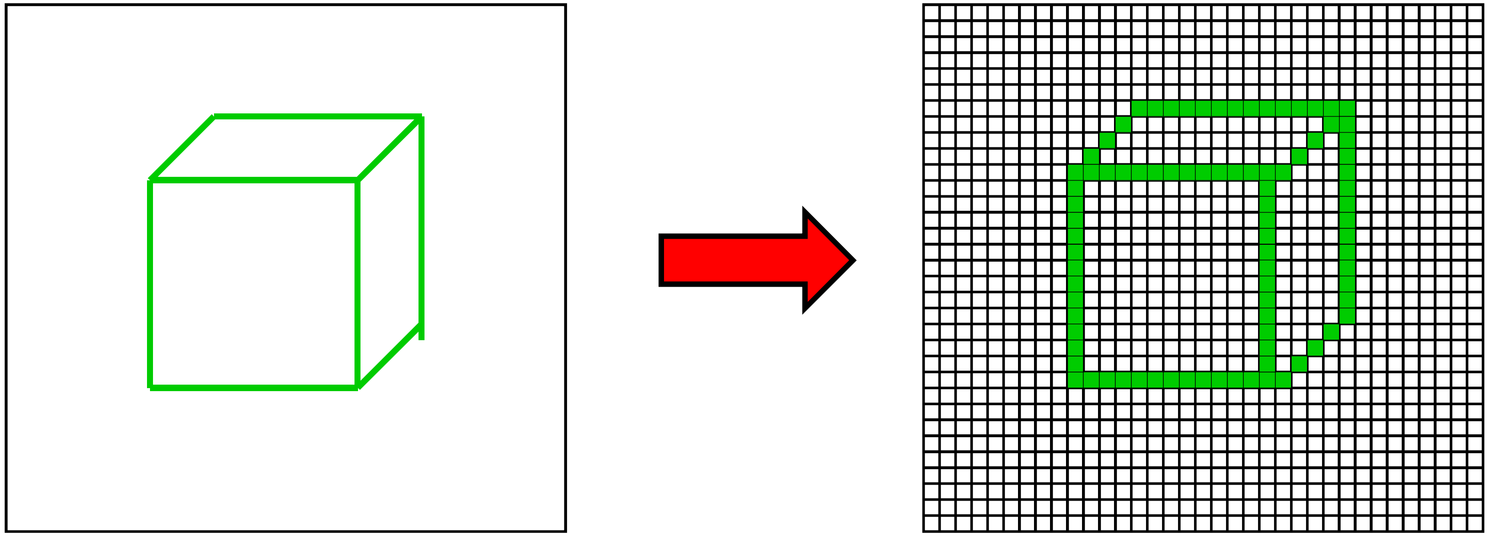
How to Draw This?



Start From Simple

How to draw a line: $y(x) = mx + b$?

Scan Conversion, a.k.a. Rasterization



Ideal Picture Raster Representation

Scan Conversion: Process of converting ideal to raster

Scan Conversion Algorithms

- A discrete set of pixels can only *approximate* a continuous geometric object
- This means that scan conversion usually introduces error
- Properties of good scan conversion algorithms:
  - *Accuracy*
  - *Efficiency*
- Challenges
  - Modify all *the right* pixels
  - Modify *only* the right pixels
  - Calculate their values correctly
  - Do it quickly
- *So, start with a correct algorithm and optimize it*

A Really Simple Line Algorithm

- Equation for a line: $y(x) = mx + b \ (0 <= x < 1)$
- Step along one pixel at a time in the "fast" direction, here x direction, fill in one pixel per column
- So, just evaluate for each x

```
void line (int x0, int y0, int x1, int y1){
    float m = whatever;
    float b = whatever;
    int x;
    for(x=x0;x<=x1;x++) {
        float y= m*x + b;
        draw_pixel(x,Round(y));
    }
}
```

- Certainly correct, but slow:
  - integer add, cast to float, floating multiply and add, plus round every step.

Lines: DDA Algorithm

- **Optimize the previous to remove multiply from inner loop.**
- **If we know y(x), we can calculate y(x+1):**

  y(x+1) = mx + m + b = y(x) + m

```
void line (int x0, int y0, int x1, int y1){
    float y = y0;
    float m = (y1 - y0)/ (float) (x1 - x0);
    int x;
    for(x=x0;x<=x1;x++) {
        draw_pixel(x,Round(y));
        y += m;
    }
}
```

- **This is called *Digital Differential Analyzer* (DDA)**
- **Problem: Floating-point add and rounds are expensive**

Bresenham's Algorithm

This does the right thing (same as DDA) at
a **cost of only 2 or 3 integer adds per point**.
(assumes sorted endpoints, 0<slope<1)

```
void draw_line(int x0, int y0, int x1, int y1) {
    int x, y = y0;
    int dx = 2*(x1-x0), dy = 2*(y1-y0);
    int dydx = dy-dx, F = dy-dx/2;

    for (x=x0 ; x<=x1 ; x++) {
        draw_pixel(x, y);
        if (F<0) F += dy;
        else {y++; F += dydx;}
    }
}
```

why does this work?

Implicit Function for a Line

Line $\mathbf{L}$ from $[x_0, y_0]$ *to* $[x_1, y_1]$.

$\mathbf{P}_0 = [x_0, y_0]$,

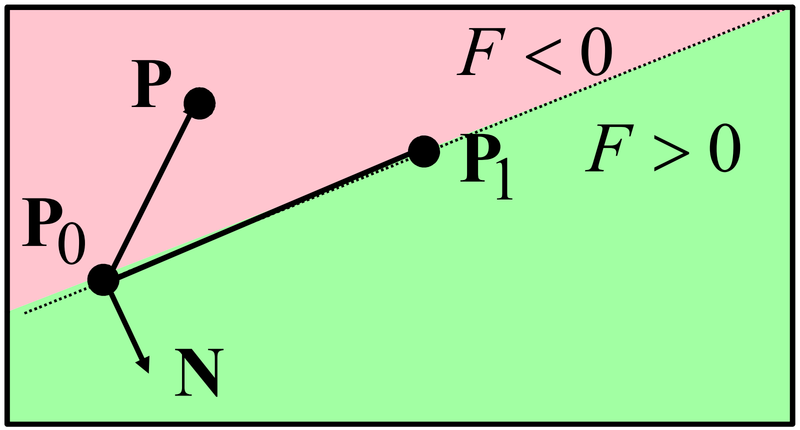
$\mathbf{P}_1 = [x_1, y_1]$.

$dx = x_1 - x_0, \; dy = y_1 - y_0$

$\mathbf{N} = [dy, -dx]$

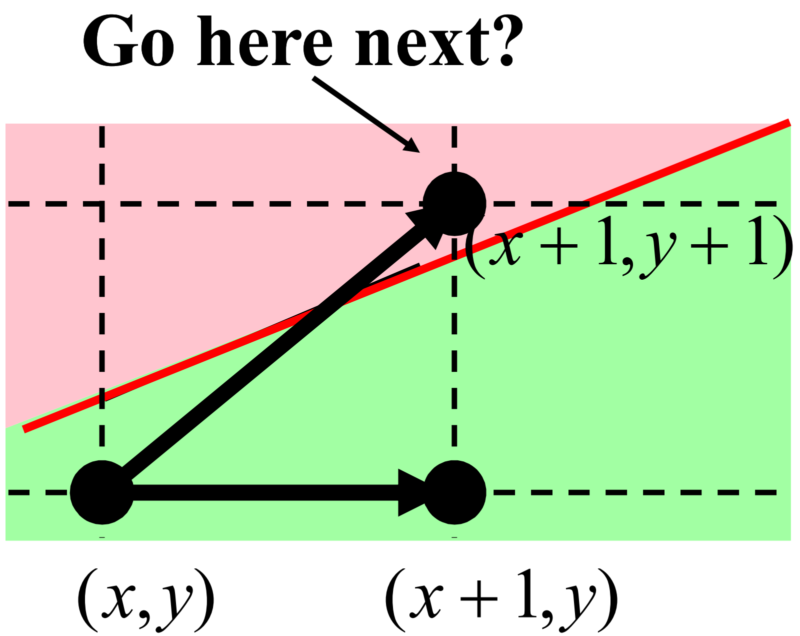
implicit function : $F(\mathbf{P}) = 2\mathbf{N} \cdot (\mathbf{P} - \mathbf{P}_0)$

$F = 0 \rightarrow \mathbf{P}$ is on $\mathbf{L}$



Why the factor of 2? Because we're going to divide by 2 later.

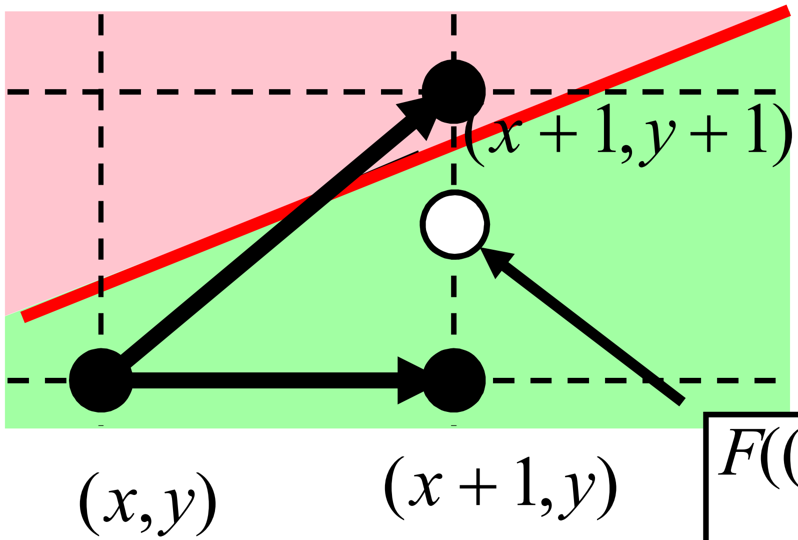
Line Drawing: Which Pixel is Next?



- Assume:
  - 0 < slope < 1
  - sorted endpoints, $x_0 < x_1$
- At each step:
  - Current point is (x, y)
  - Next point is pixel (x+1, ?) that's closest to the actual line
  - Do we increment x and y or only x?
- Use the implicit function to decide!

Use the Implicit Function

- Idea: Test the half-way point (x+1, y+1/2)



$F((x+1, y+1/2)) > 0$?

yes: increment x and y

no: increment x

Trick: Incrementally Update *F*

$$\mathbf{P} = (x, y), \Delta = (1, 1/2)$$

$$F(\mathbf{P}) = \mathbf{N} \cdot (\mathbf{P} - \mathbf{P}_0)$$

$$F(\mathbf{P} + \Delta) = \mathbf{N} \cdot (\mathbf{P} + \Delta - \mathbf{P}_0)$$

$$= F(\mathbf{P}) + \mathbf{N} \cdot \Delta$$

- What we care about here is only the sign of F, so multiply the function by 2 to avoid floating point calculation

Trick: Incrementally Update *F*

$$F(\mathbf{P}) = 2\mathbf{N} \cdot (\mathbf{P} - \mathbf{P}_0)$$

$$F(\mathbf{P}+\Delta) = 2\mathbf{N} \cdot (\mathbf{P}+\Delta-\mathbf{P}_0)$$

$$= F(\mathbf{P}) + 2\mathbf{N} \cdot \Delta$$

- Computing F(P) requires a dot product:
  - 2 multiplications and 1 add
- But computing F(P+Δ) requires only 1 add
  - The 2N• Δ term is constant – it only needs to be calculated once
- Δ is [1,0] or [1,1]

Decision Variable F

$$F_0 = F(\mathbf{P}_0 + [1, 1/2])$$
$$= F(\mathbf{P}_0) + \mathbf{N} \cdot [2,1]$$

$$\mathbf{N} = [dy, -dx]$$

$$F = F + 2\mathbf{N} \cdot \Delta$$

where

$$\Delta = [1,0] \text{ or } [1,1]$$

i.e.,

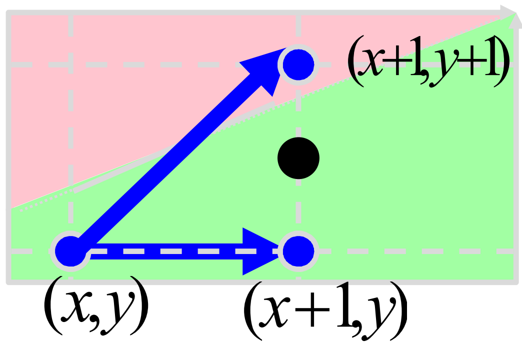
$$F = F(P_0) + 2dy - dx$$

$$\mathit{If}\ F<0 \quad F = F + 2dy$$

or

$$\mathit{If}\ F>=0 \quad F = F + 2\,dy - 2dx$$

- Initialize *x*, *y*, F
- Loop until end of line:
  - draw pixel *(x, y)*
  - increment *x*
  - if F>0, increment *y*
  - increment F according to whether Δ is [1, 0] or [1, 1]



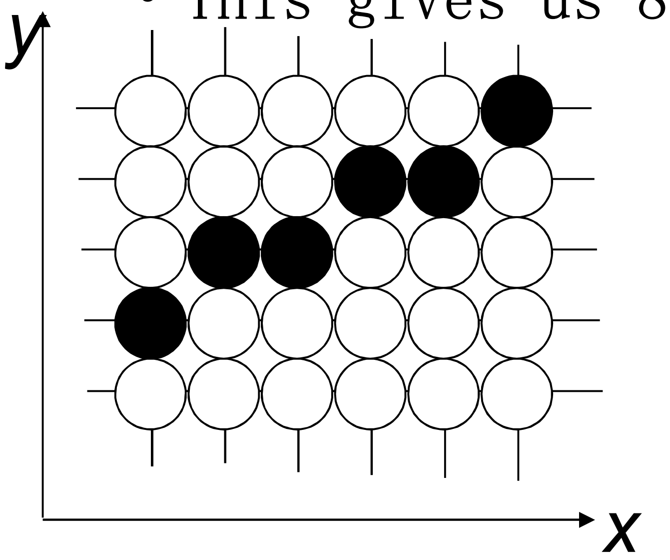
Bresenham Line Algorithm

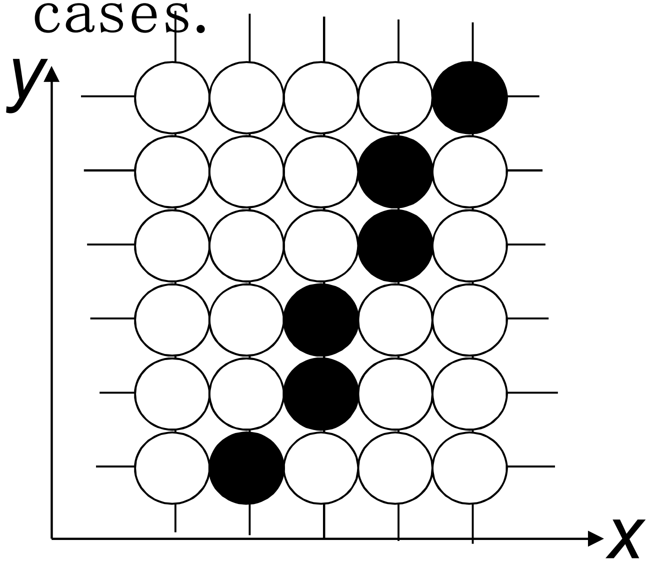
This does the right thing (same as DDA) at a **cost of only 2 or 3 integer adds per point**.
(assumes sorted endpoints, 0<slope<1)

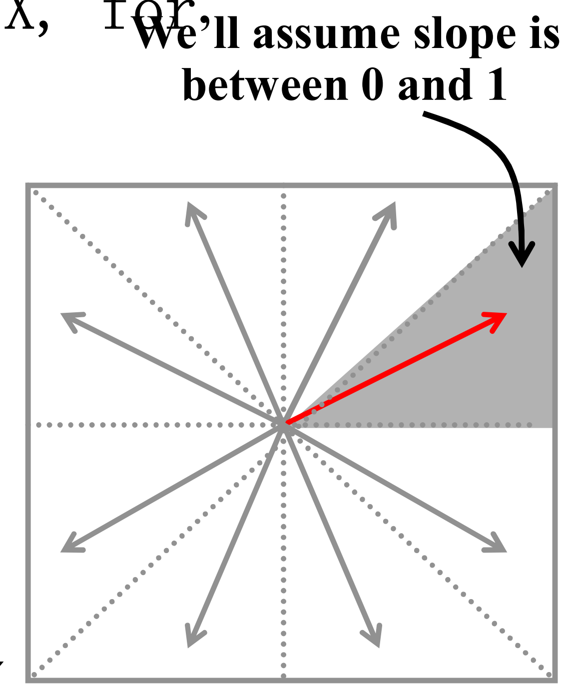
```
void draw_line(int x0, int y0, int x1, int y1) {
    int x, y = y0;
    int dx = 2*(x1-x0), dy = 2*(y1-y0);
    int dydx = dy-dx, F = dy-dx/2;

    for (x=x0 ; x<=x1 ; x++) {
        draw_pixel(x, y);
        if (F<0) F += dy;
        else {y++; F += dydx;}
    }
}
```

Line Drawing, Cases by Octant

- The algorithms for drawing lines need to step along one pixel at a time in the "fast" direction, which depends on the slope of the line
- We also have to worry about reversed end point order (drawing from large to small X, for example).
- This gives us 8 cases.

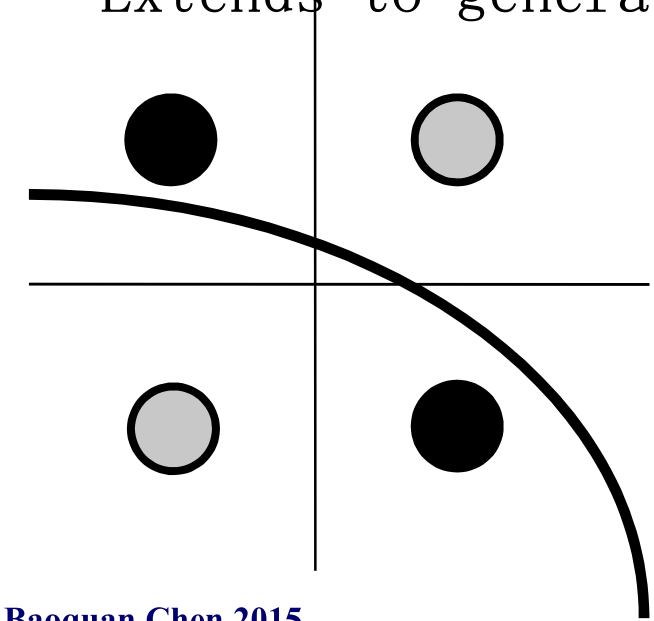


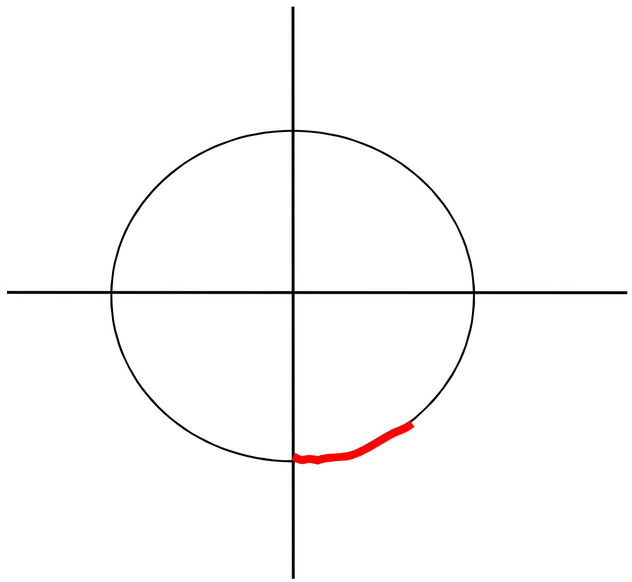




Bresenham Algorithm for Circles

- Same approach as line algorithm
  - use a decision variable formula derived for a circle ($F = x^2 + y^2 - r^2$)
- Eightfold symmetry
  - only compute the points for one octant – use sign flips to give the rest
- Extends to general conics (ellipses...)





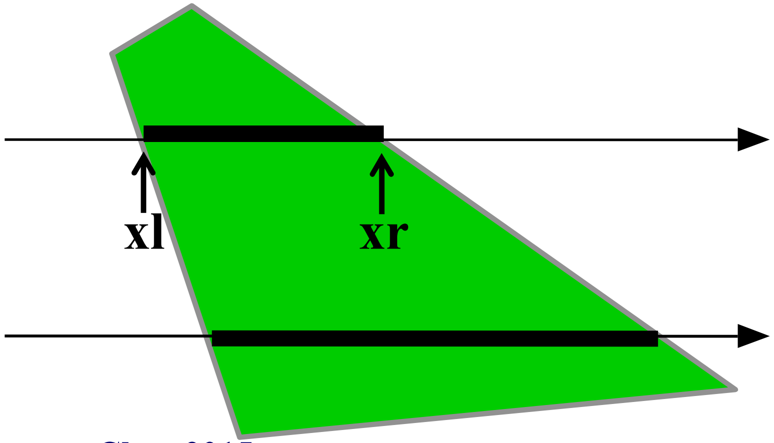
Bresenham Circle Algorithm

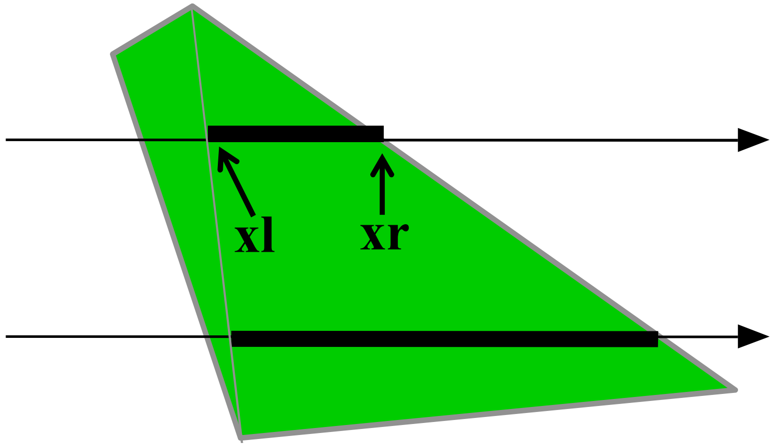
This draws a circle by calculating in one octant
and re-using the resulting point 8 times

```
void draw_circle(int radius) {
   int x = 0, y = radius;
   int d = 1-radius;
   while (y>x) {
      if (d<0) /* select East point next */
         d += 2*x + 3;
      else {    /* select South-East point next */
         d += 2*(x-y) + 5;
         y--;
      }
      x++;
      draw_8_pts(x,y); /* draws point in each octant */
   }
}
```

Scan Converting Filled, Convex Polygons

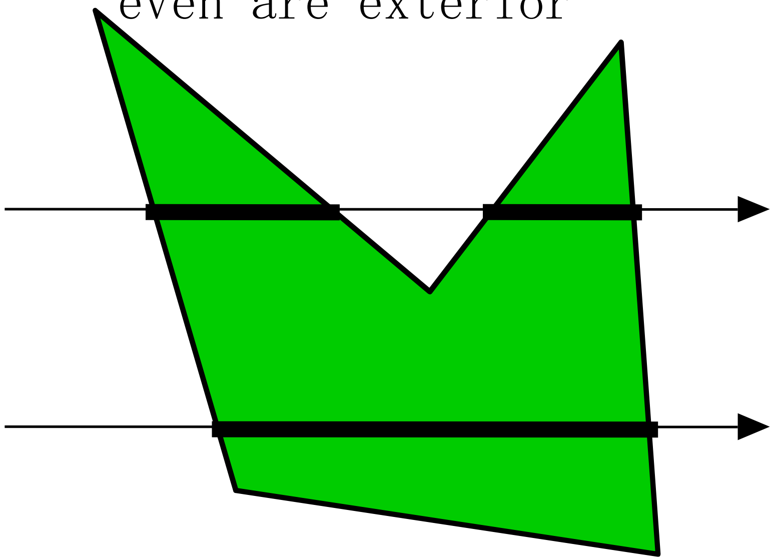
- Find top and bottom vertices
- Make list of edges along left and right sides
- For each scanline from top to bottom
  - There's a single span to fill
  - Find left & right endpoints of span, xl & xr, (can use Bresenham's algorithm
  - Fill pixels inbetween xl & xr
- If you don't do all of the above carefully, cracks or overlaps between abutting polygons result!



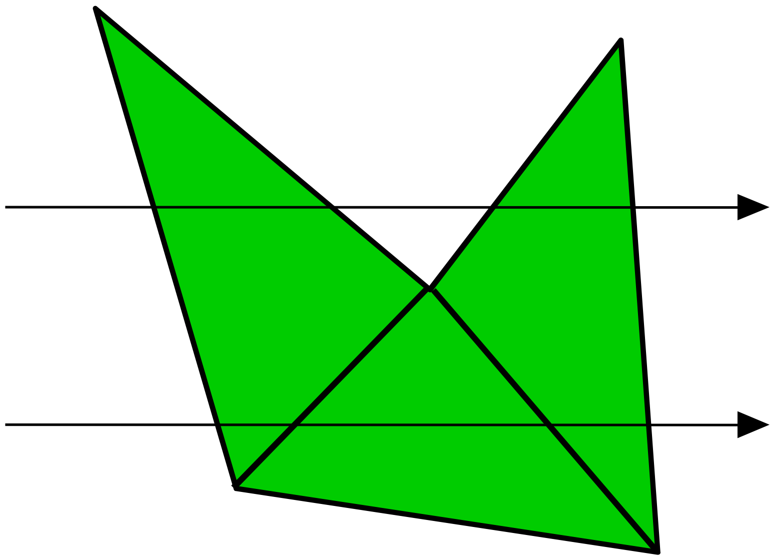


Scan Converting Filled, Concave Polygons

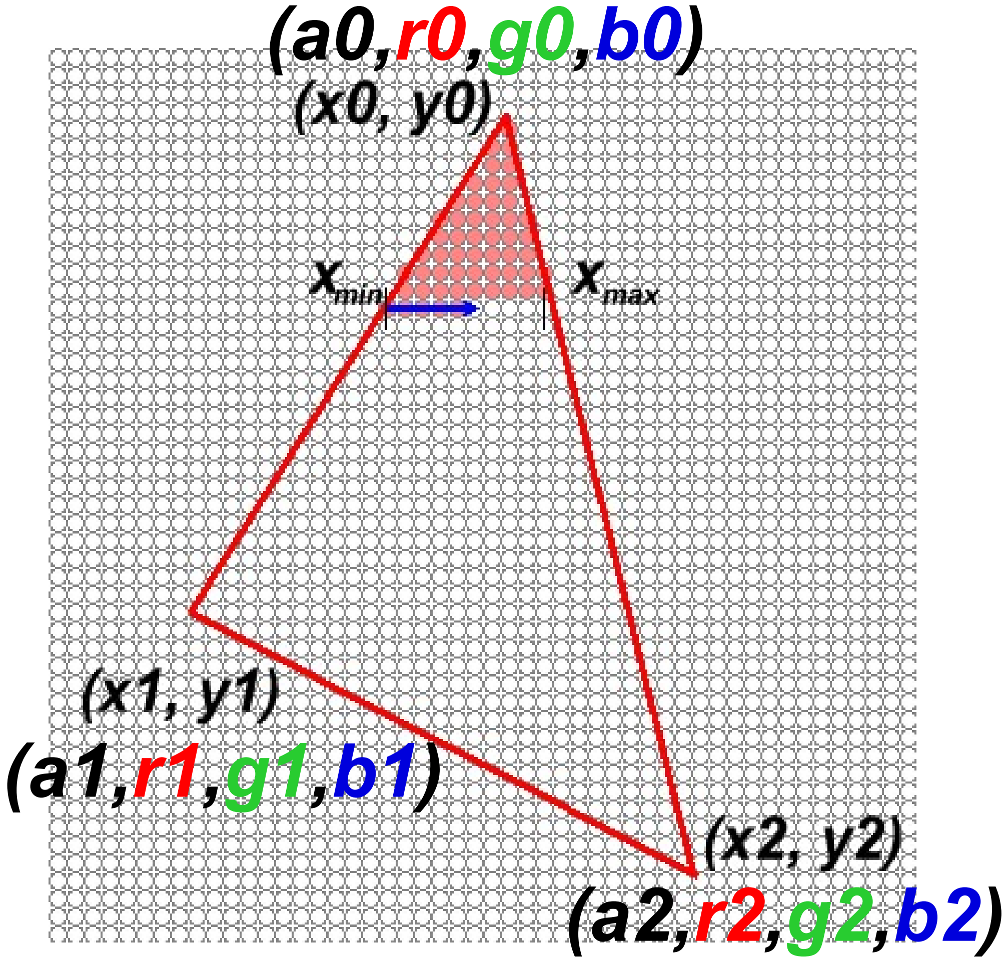
- For each scanline
  - Find all the scanline/polygon intersections
  - Sort them left to right
  - Fill the interior *spans* between intersections
  - *Parity Rule:* odd ones are interior, even are exterior



- Or, triangulation

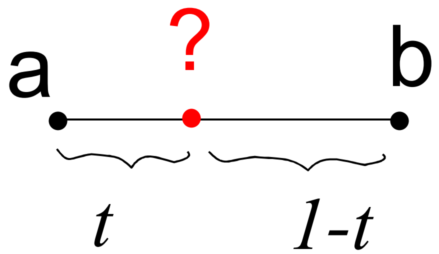


Color Interpolations



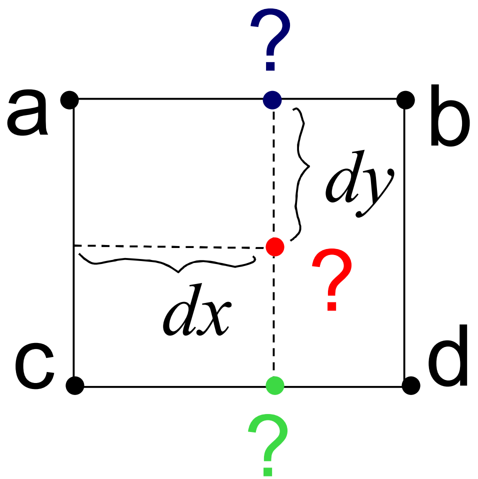
Review on Interpolation

- **Linear Interpolation**



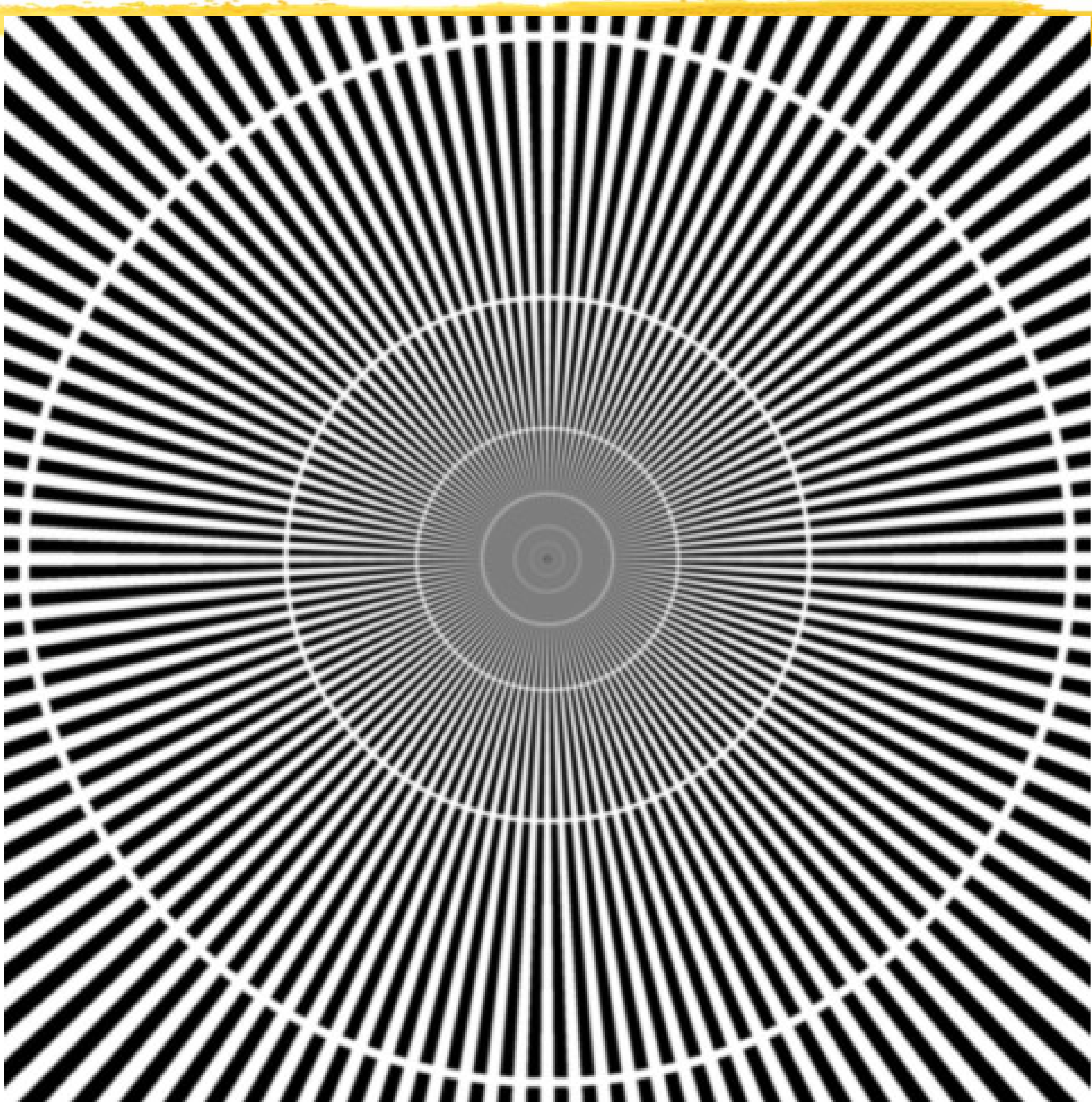
$$? = a(1-t)+bt$$
$$= a + (b-a)t$$

- **Bilinear Interpolation**



$$? = a(1-dx)+bdx$$
$$? = c(1-dx)+ddx$$
$$? = ?(1-dy)+?dy = ? + (?-?)dy$$
$$= a(1-dx)(1-dy)+bdx(1-dy)$$
$$+c(1-dx)dy+ddxdy$$

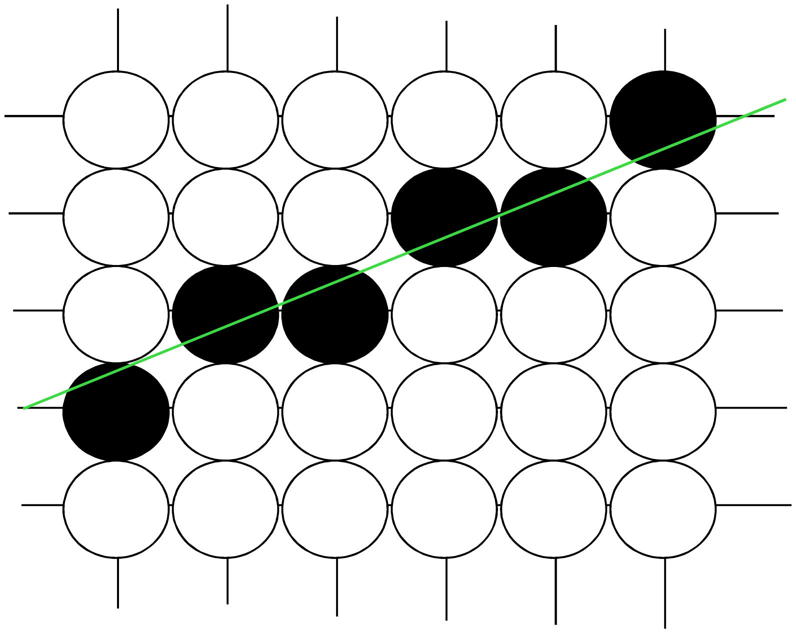
Again, How to Draw This?

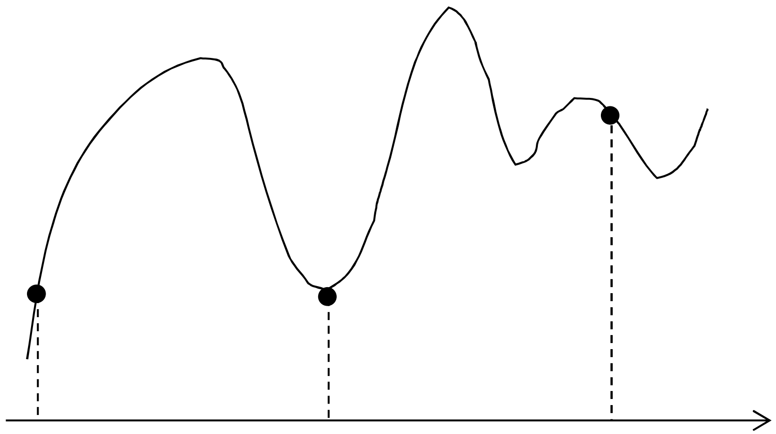


Aliasing

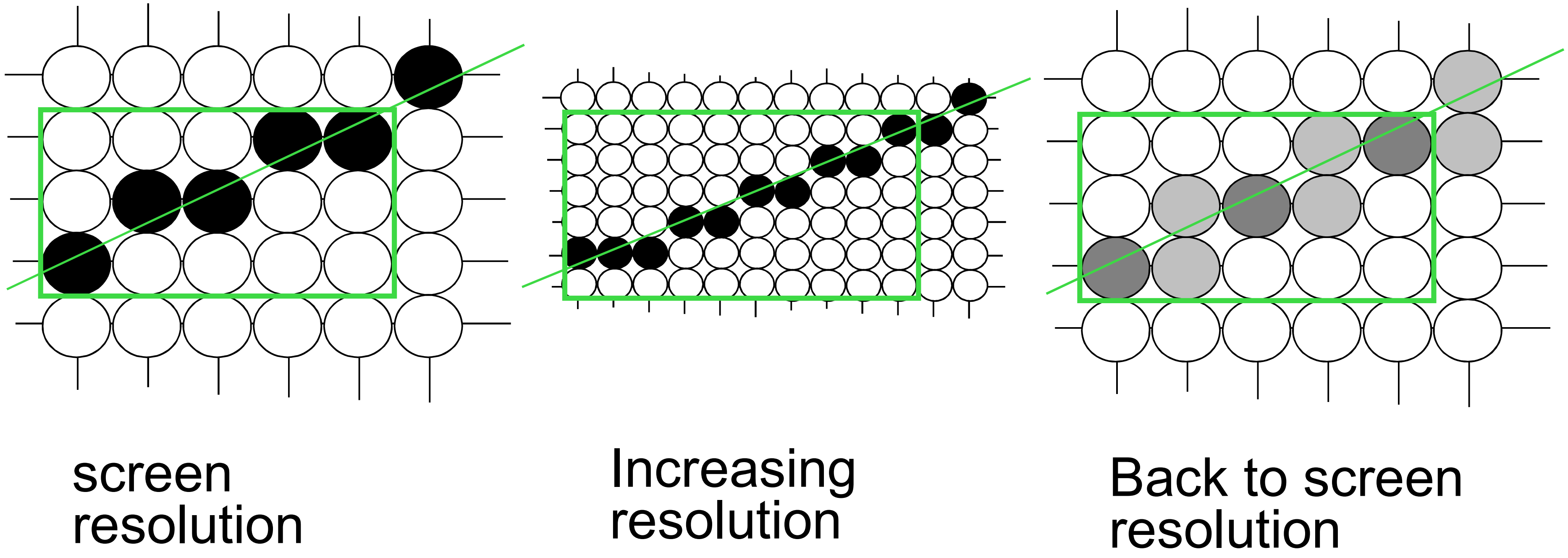
Samples don't capture geometry changes

- Not dense enough!

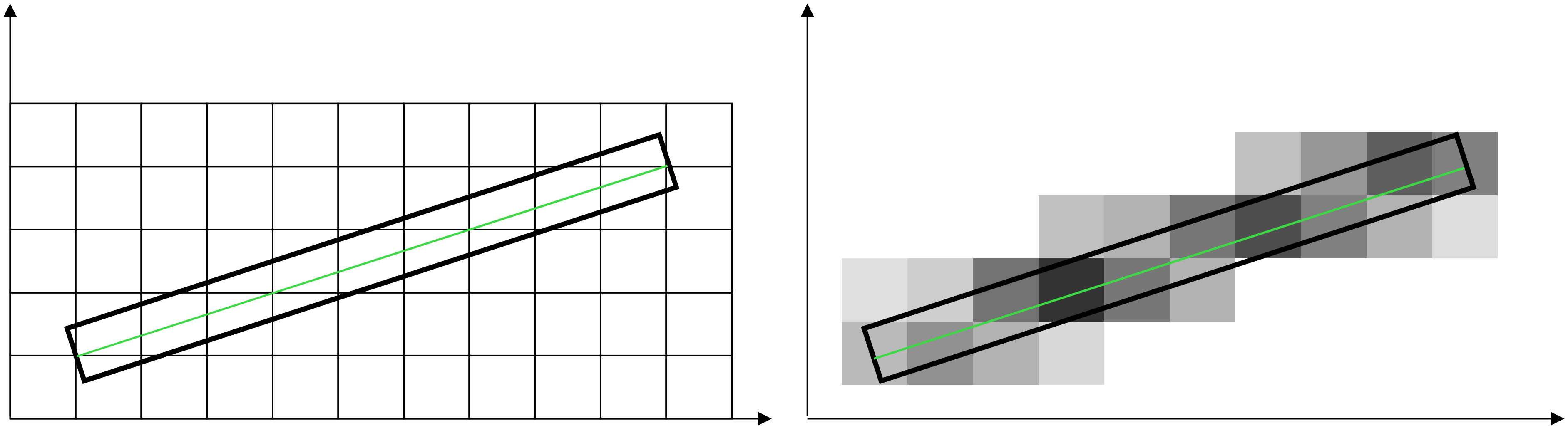




Antialiasing: Super-sampling

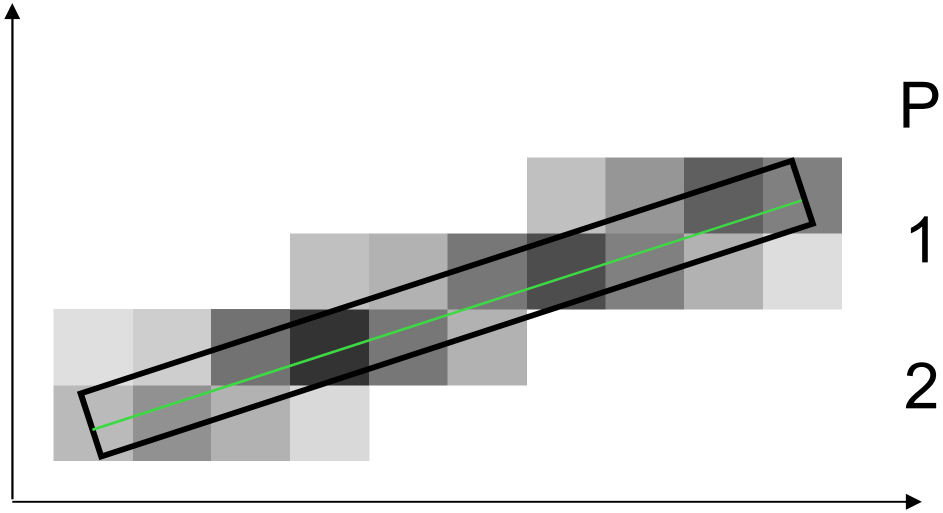


Antialiasing: Unweighted Area Sampling



- Line with ‘thickness’
- Pixel’s color, here ‘blackness’, depends on the intersection area between the thick line and the pixel square

Antialiasing: Unweighted Area Sampling



Properties:

1. Intenstity soley based on intersection area
2. Equal areas equal intensity ?

However, the same area closer to the pixel center should have greater influence than does one at a greater distance! This consideration leads to

Weighted Area Sampling:

‘blackness’ = area*f(distance),

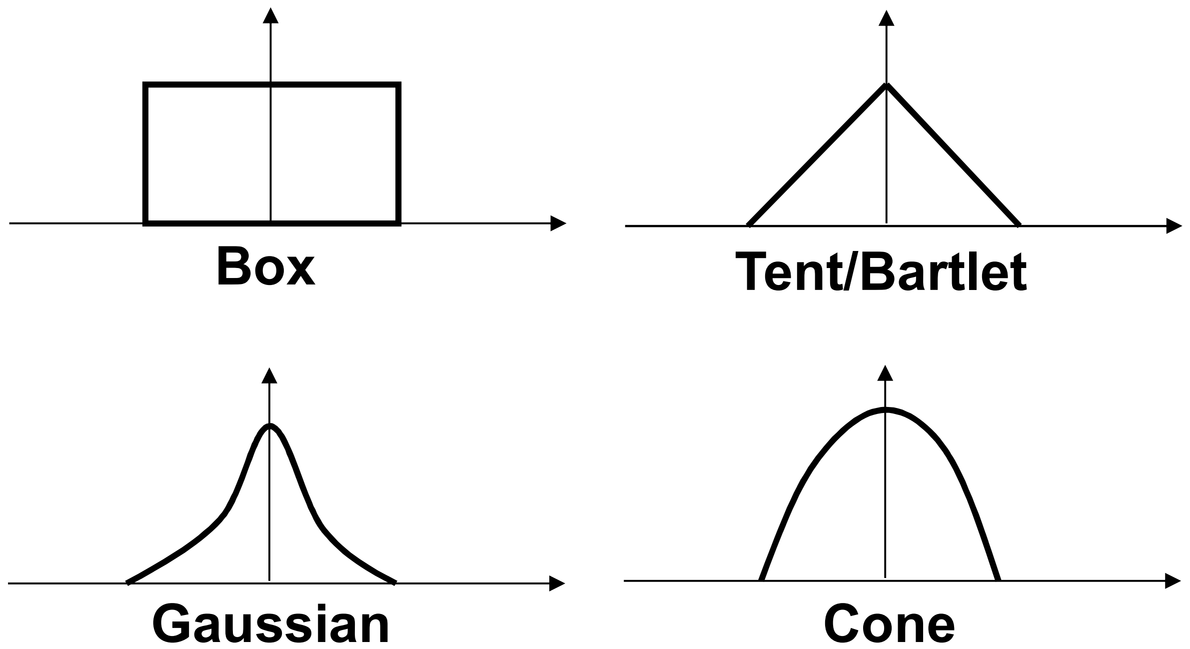
Where f: weighting function, dist: pixel center distance to the line

Antialiasing: Weighted Area Sampling

We can define many weighting functions!

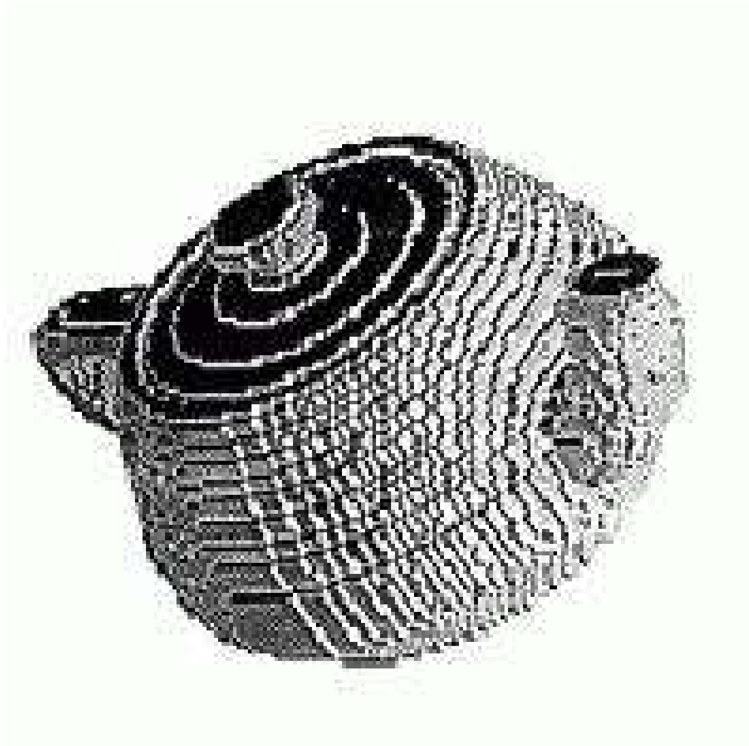
Can be anything, BUT,

1. Finite non-zero region
2. Meaningful (e.g., decreasing from high to zero value when distance increases
3. Full ‘area’ equal to 1



Extend Everything to 3D

Voxelization

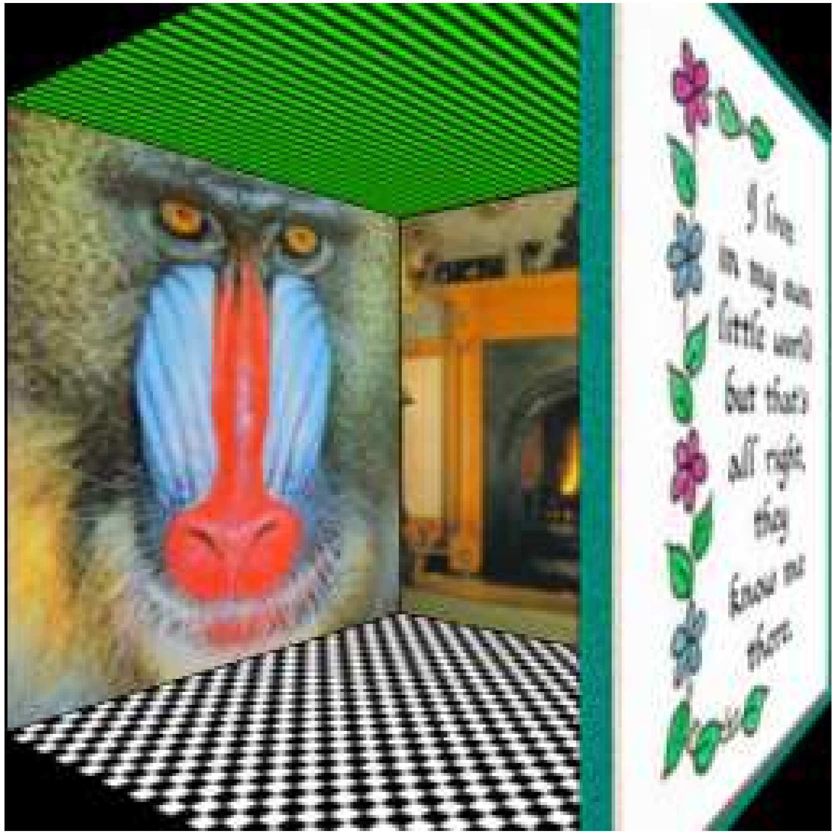


More Issues

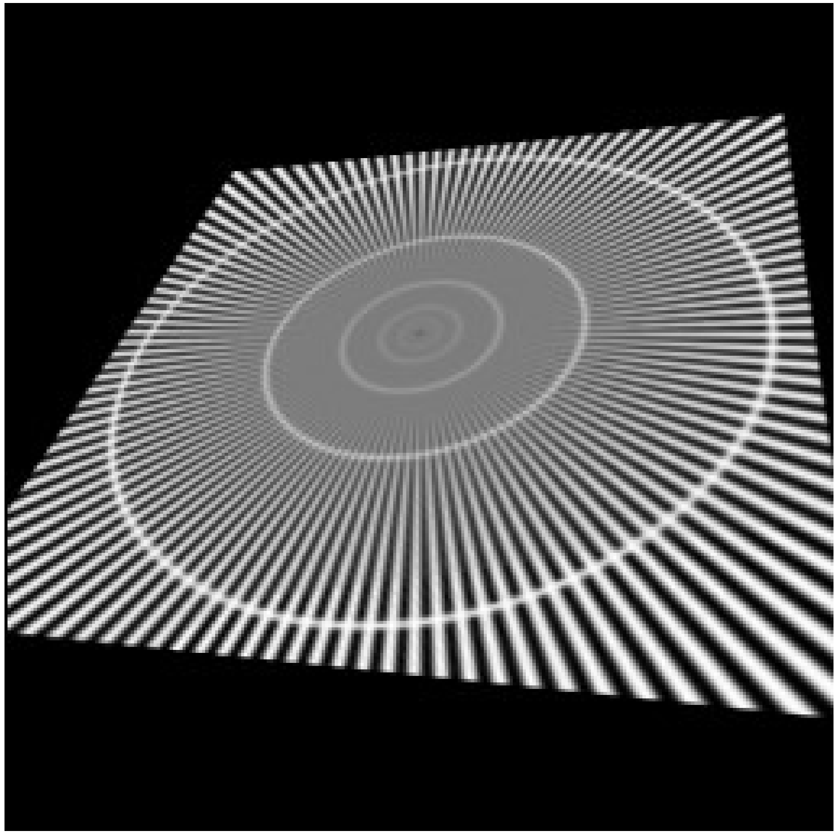
The devils in the details:

1. Non-integer endpoints
   (occurs frequently when rendering 3D lines)
2. What if a line endpoint lies outside the viewing area?
3. How do you handle thick lines?
4. Optimizations for connected line segments
5. Lines show up in the strangest places
6. ……

My Previous Works



http://www.cs.sdu.edu.cn/~baoquan/papers/fast.pdf



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